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MATHEMATICAL MODELING AND AVAILABILITY OPTIMIZATION OF EMBEDDED LIFE CRITICAL SYSTEMS*

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Abstract. The prominent objective of present study is to propose a novel stochastic model for availability optimization of the embedded life critical systems. Embedded life critical systems comprise using software and hardware components and human being works as an operator. In present analysis provision of redundant software and hardware components is made. The system may suffer due to software, hardware, and operator failures. A trained repairman always remains available to upgrade software and repair the hardware. The operator undergoes for treatment upon failure. All time dependent random variables are exponentially distributed. The availability function is derived using Markov birth-death process and optimized by well-known algorithms Dragon Fly (DA) and Gray Wolf optimization (GWO) algorithms. It is observed that GWO outperforms the DA algorithm in terms of elapsed time and convergence rate.

Keywords: Embedded life critical systems, availability, optimization, Markov process.
AMS Subject Classification: 90B25, 60K10.
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1 Introduction

Embedded systems extensively used in most of the sectors and are designed using one or more processors for execution of a specific task. As technology advances, the narrowing of the gap between embedded systems and other computing devices and systems designed with multiple features has been visualized, but many systems, such as smartphone processing units, are still embedded. Some of the major challenges faced by system designers are processor selection, product enhancements, and rapid product delivery. Embedded systems consist of hardware and software components. Hardware and software requirements vary from system to system, depending on their applicability and manufacturing goals. Compared to other computer systems, embedded systems are subject to stricter restrictions. The different applications of embedded systems and the different requirements of customers present challenges for system designers in designing generalized models of such systems. This allows system designers to study hardware and software together when drawing design conclusions. However, this can pose some issues in terms of size, cost, design space, and complexity. Advances in technology have also established a new generation of devices that improve performance. However, there can be some challenges. Therefore, it is important to evaluate the reliability of these systems. Several approached have been proposed by researchers to increase the availability of the system models. Redundancy is

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one of the key methods of reliability and availability improvement. In many sectors like medical, transport, communication and industrial redundancy is extensively used. Many researchers like Wattanapongsakorn and Levitan (2004), Yadavalli et al. (2005), Huang and Chang (2007), Brooks et al. (2007), El-Said Khaled and El-Sherbeny Mohamed (2010), Meedeniya et al. (2011), Singh and Gupta (2012), El-Sherbeny (2017), Kumar and Saini (2018), Neggaz et al. (2018), and Saini and Kumar (2020) proposed several models under various set of assumption for reliability evaluation o the systems. But most of these studies concentrated on the derivation of the local solution of the problem. In these days, various metaheuristic approaches exist which provides the global solution of the problem. Jagtap et al. (2021) optimized availability of thermal power plant water circulation system using PSO. Kumar et al. (2022) proposed an efficient stochastic model for availability optimization of cooling tower using metaheuristic algorithms GA and PSO. It is identified that PSO provides better results than GA in availability prediction of cooling towers. Saini et al. (2022) developed a novel stochastic model for availability optimization of condenser of power plants using GA and PSO. Recently, many swarm intelligence-based algorithms has been developed for availability optimization of industrial systems. Mirjalili et al. (2014) proposed gray wolf optimizer and Mafarja et al. (2020) presented the Dragonfly algorithm for optimization of reliability measures of industrial systems. Yahia et al. (2020) developed a hybrid optimization algorithm by combination of ant colony and neighbour joining methods and provide a solution for traveling salesman problem. Taj and Rizwan (2022) investigated the reliability of a 3-unit parallel system under single repairman facility. Though these algorithms rarely used in the performance optimization of embedded systems. As embedded systems extensively used in medical sector like electronic stethoscopes, MRI, PET scan, CT scan, glucose monitors, pacemakers, and CPAP machines are few examples of embedded systems. So, assurance of reliability and availability of these systems is necessary without any cost restriction to save the human life. Hence the present work is motivated by extensive use of embedded systems in medical sector.

Claims of Study:
Till now very few efforts has been made for availability evaluation of embedded systems under concept of redundancy and controller failure. Using Markov process state transition model is developed as shown in fig. 1 for availability evaluation and swarm intelligence-based algorithms are employed to obtain the optimal availability. Researchers claimed the novelty of the proposed system based on below mentioned points:

- **Design:** The concept of cold standby redundancy is utilized in the development of model. The redundant component is used for software as well as hardware. The idea of failure of controller is not so far discussed in embedded systems.
- **Availability Optimization:** In our model availability of the embedded system is predicted by using Dragonfly and Gray wolf optimization algorithms and it is observed that Gray Wolf algorithm outperforms the Dragon fly algorithm.
- **Elapsed Time:** It is observed that Gray Wolf algorithm takes less time in the execution of the objective function in comparison of Dragon fly algorithm.

2 System description

Embedded life critical systems comprise using software and hardware components and human being works as an operator. In present analysis provision of redundant software and hardware components is made. The system may suffer due to software, hardware, and operator failures. A trained repairman always remains available to upgrade software and repair the hardware. The operator undergoes for treatment upon failure. All time dependent random variables are exponentially distributed. The availability function is derived using Markov birth-death process and optimized by well-known algorithms Dragon Fly (DA) and Gray Wolf optimization (GWO) algorithms. In proposed system, software (A), hardware (B) and operator (C) are involved as well as provision of software and hardware unit is also made. The system model is shown in Figure 1.

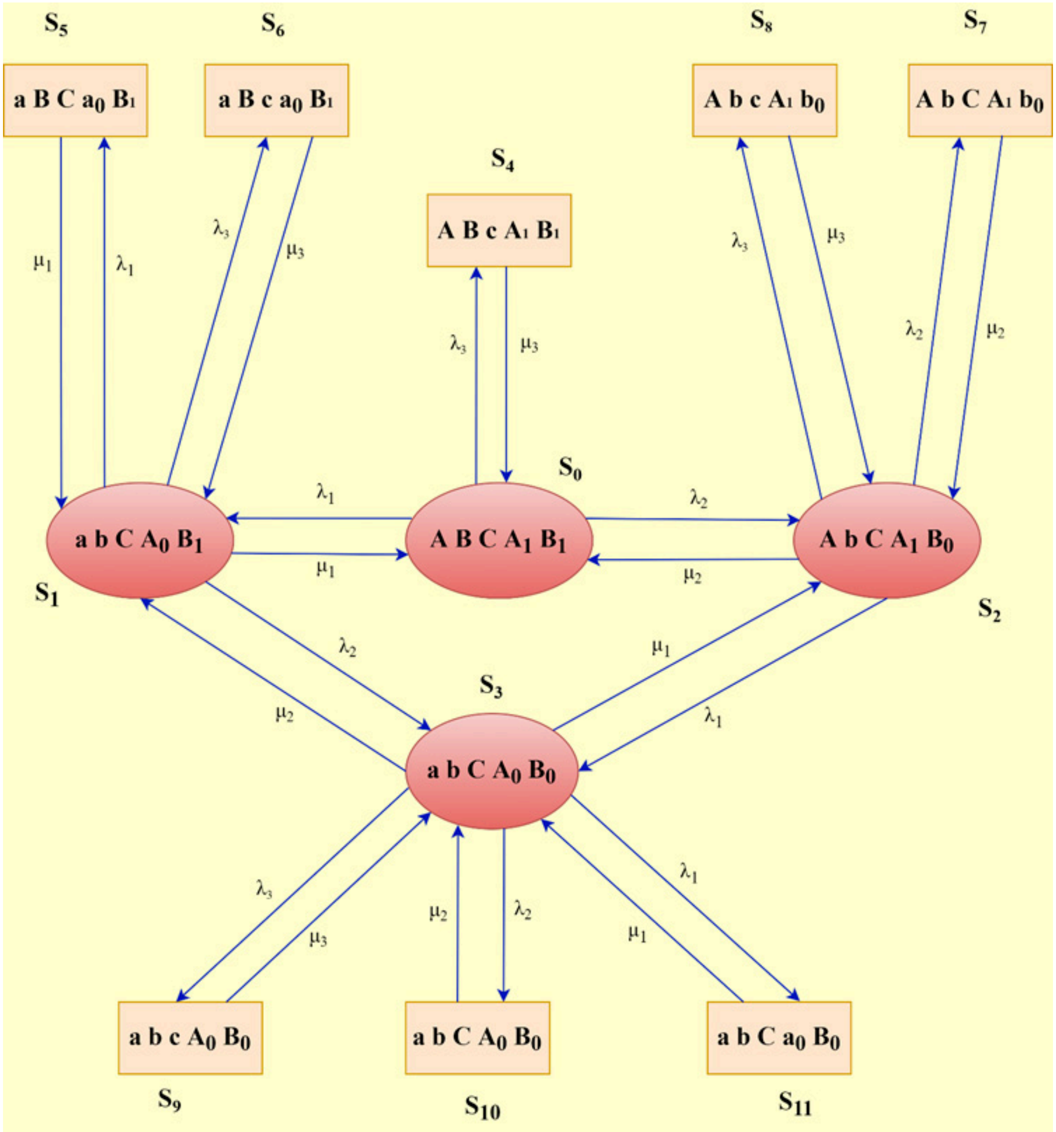

Figure 1: State transition diagram of Embedded life critical system

3 Mathematical modeling of embedded life critical system

Here, using Markov birth death process, mathematical model of the embedded system is developed based on state transition diagram Figure 1. At any time, t, if embedded system is in state S_k, then the probability that the system to be in state k is defined as: Probability that system is in state k at time t and remain there in time interval $(t, t + \Delta t)$ or if it is at any other state at time t then it transit to state S_k, in time interval $(t, t + \Delta t)$ provided transition exist between the states and $\Delta t \to 0$. Using the same concept the differential-difference equation at state S_0 is derived as follows:

$$\begin{aligned} P_0(t+\Delta t) &= (1-\lambda_1\Delta t-\lambda_2\Delta t-\lambda_3\Delta t)P_0(t)+\mu_1P_1(t)\Delta t+\mu_2P_2(t)\Delta t+\mu_3P_4(t)\Delta t \\ &= P_0(t)-(\lambda_1\Delta t+\lambda_2\Delta t+\lambda_3\Delta t)P_0(t)+\mu_1P_1(t)\Delta t+\mu_2P_2(t)\Delta t+\mu_3P_4(t)\Delta t \end{aligned}$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3)P_0(t)+\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_4(t)$$
$$P_0'(t)=-(\lambda_1+\lambda_2+\lambda_3)P_0(t)+\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_4(t)\,.$$

Taking limit $\lim\limits_{t\to\infty}$, we get

$$\lim_{t\to\infty}P_0'(t)=-(\lambda_1+\lambda_2+\lambda_3)P_0(t)+\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_4(t)$$
$$\mu_1P_1+\mu_2P_2+\mu_3P_4-(\lambda_1+\lambda_2+\lambda_3)P_0=0 \tag{1}$$

The differential-difference equations of rest of the states are derived below:

$$P_1(t+\Delta t)=(1-\lambda_1\Delta t-\lambda_2\Delta t-\lambda_3\Delta t-\mu_1\Delta t)P_1(t)+\mu_1P_5(t)\Delta t+\mu_3P_6(t)\Delta t+\mu_2P_3(t)\Delta t+\lambda_1P_0(t)\Delta t$$
$$P_1(t+\Delta t)=P_1(t)-(\lambda_1+\lambda_2+\lambda_3+\mu_1)P_1(t)\Delta t+\mu_1P_5(t)\Delta t+\mu_3P_6(t)\Delta t+\mu_2P_3(t)\Delta t+\lambda_1P_0(t)\Delta t$$
$$\lim_{\Delta t\to 0}\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3+\mu_1)P_1(t)+\mu_1P_5(t)+\mu_3P_6(t)+\mu_2P_3(t)+\lambda_1P_0(t)\Delta t$$
$$P_1'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_1)P_1(t)+\mu_1P_5(t)+\mu_3P_6(t)+\mu_2P_3(t)+\lambda_1P_0(t)$$
$$\lim_{t\to\infty}P_1'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_1)P_1(t)+\mu_1P_5(t)+\mu_3P_6(t)+\mu_2P_3(t)+\lambda_1P_0(t)$$
$$-(\lambda_1+\lambda_2+\lambda_3+\mu_1)P_1+\mu_1P_5+\mu_3P_6+\mu_2P_3+\lambda_1P_0=0 \tag{2}$$
$$P_2(t+\Delta t)=(1-\lambda_1\Delta t-\lambda_2\Delta t-\lambda_3\Delta t-\mu_2\Delta t)P_2(t)+\mu_1P_3(t)\Delta t+\mu_3P_8(t)\Delta t+\mu_2P_7(t)\Delta t+\lambda_2P_0(t)\Delta t$$
$$P_2(t+\Delta t)=P_2(t)-(\lambda_1+\lambda_2+\lambda_3+\mu_2)P_2(t)\Delta t+\mu_1P_3(t)\Delta t+\mu_3P_8(t)\Delta t+\mu_2P_7(t)\Delta t+\lambda_2P_0(t)\Delta t$$
$$\lim_{\Delta t\to 0}\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3+\mu_2)P_2(t)+\mu_1P_3(t)+\mu_3P_8(t)+\mu_2P_7(t)+\lambda_2P_0(t)$$
$$P_2'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_2)P_2(t)+\mu_1P_3(t)+\mu_3P_8(t)+\mu_2P_7(t)+\lambda_2P_0(t)$$
$$\lim_{t\to\infty}P_2'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_2)P_2(t)+\mu_1P_3(t)+\mu_3P_8(t)+\mu_2P_7(t)+\lambda_2P_0(t)$$
$$-(\lambda_1+\lambda_2+\lambda_3+\mu_2)P_2+\mu_1P_3+\mu_3P_8+\mu_2P_7+\lambda_2P_0=0 \tag{3}$$
$$P_3(t+\Delta t)=(1-\lambda_1\Delta t-\lambda_2\Delta t-\lambda_3\Delta t-\mu_1\Delta t-\mu_2\Delta t)P_3(t)+\mu_1P_{11}(t)\Delta t+\mu_2P_{10}(t)\Delta t+\mu_3P_9(t)\Delta t+\lambda_1P_2(t)\Delta t+\lambda_2P_1(t)\Delta t$$
$$P_3(t+\Delta t)=P_3(t)-(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2)P_3(t)\Delta t+\mu_1P_{11}(t)\Delta t+\mu_2P_{10}(t)\Delta t+\mu_3P_9(t)\Delta t+\lambda_1P_2(t)\Delta t+\lambda_2P_1(t)\Delta t$$
$$\lim_{\Delta t\to 0}\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2)P_3(t)+\mu_1P_{11}(t)+\mu_2P_{10}(t)+\mu_3P_9(t)+\lambda_1P_2(t)+\lambda_2P_1(t)$$
$$P_3'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2)P_3(t)+\mu_1P_{11}(t)+\mu_2P_{10}(t)+\mu_3P_9(t)+\lambda_1P_2(t)+\lambda_2P_1(t)$$
$$\lim_{t\to\infty}P_3'(t)=-(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2)P_3(t)+\mu_1P_{11}(t)+\mu_2P_{10}(t)+\mu_3P_9(t)+\lambda_1P_2(t)+\lambda_2P_1(t)$$
$$-(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2)P_3+\mu_1P_{11}+\mu_2P_{10}+\mu_3P_9+\lambda_1P_2+\lambda_2P_1=0 \tag{4}$$
$$P_4(t+\Delta t)=(1-\mu_3\Delta t)P_4(t)+\lambda_3P_0(t)\Delta t$$
$$P_4(t+\Delta t)=P_4(t)-\mu_3P_4(t)\Delta t+\lambda_3P_0(t)\Delta t$$
$$\lim_{\Delta t\to 0}\frac{P_4(t+\Delta t)-P_4(t)}{\Delta t}=-\mu_3P_4(t)+\lambda_3P_0(t)$$
$$P_4'(t)=-\mu_3P_4(t)+\lambda_3P_0(t)$$
$$\lim_{t\to\infty}P_4'(t)=-\mu_3P_4(t)+\lambda_3P_0(t)$$

$$-\mu_3 P_4 + \lambda_3 P_0 = 0 \tag{5}$$
$$-\mu_1 P_5 + \lambda_1 P_1 = 0 \tag{6}$$
$$-\mu_3 P_6 + \lambda_1 P_1 = 0 \tag{7}$$
$$-\mu_2 P_7 + \lambda_2 P_2 = 0 \tag{8}$$
$$-\mu_3 P_8 + \lambda_3 P_2 = 0 \tag{9}$$
$$-\mu_3 P_9 + \lambda_3 P_3 = 0 \tag{10}$$
$$-\mu_2 P_{10} + \lambda_2 P_3 = 0 \tag{11}$$
$$-\mu_1 P_{11} + \lambda_1 P_3 = 0 \tag{12}$$

The initial conditions are as follows:

$$\begin{aligned} &P_0(0) = 1 \\ &P_i(0) = 0, \quad i = 1 \text{ to } 11 \end{aligned} \tag{13}$$

From equations (1)-(13), the expression for steady state probabilities is derived as follows:

$$\begin{aligned}
P_0 &= \frac{\mu_1 P_1 + \mu_2 P_2 + \mu_3 P_4}{\lambda_1 + \lambda_2 + \lambda_3} \\
P_1 &= \frac{\lambda_1 P_0 + \mu_2 P_3 + \mu_1 P_5 + \mu_3 P_6}{\lambda_1 + \lambda_2 + \lambda_3 + \mu_1} \\
P_2 &= \frac{\lambda_2 P_0 + \mu_1 P_3 + \mu_2 P_7 + \mu_3 P_8}{\lambda_1 + \lambda_2 + \lambda_3 + \mu_2} \\
P_3 &= \frac{\lambda_1 P_2 + \lambda_2 P_1 + \mu_1 P_{11} + \mu_2 P_{10} + \mu_3 P_9}{\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_2} \\
P_4 &= \frac{\lambda_3 P_0}{\mu_3},\ P_5 = \frac{\lambda_1 P_1}{\mu_1},\ P_6 = \frac{\lambda_3 P_1}{\mu_3},\ P_7 = \frac{\lambda_2 P_2}{\mu_2},\ P_8 = \frac{\lambda_3 P_2}{\mu_3} \\
P_9 &= \frac{\lambda_3 P_3}{\mu_3},\ P_{10} = \frac{\lambda_2 P_3}{\mu_2} \ \text{ and } \ P_{11} = \frac{\lambda_1 P_3}{\mu_1}
\end{aligned}$$

By probabilistic arguments, it is know that the sum of all transition probabilities are equal to 1, i.e., $\sum_{i=0}^{11} P_i = 1$. It implies

$$P_0 = \left[1 + \left(1 + \frac{\lambda_1}{\mu_1}\right)\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_2}{\mu_2}\frac{\lambda_2}{\mu_2} + \frac{\lambda_2}{\mu_2}\frac{\lambda_3}{\mu_3}\right)\left(\frac{\lambda_1}{\mu_1}\frac{\lambda_1}{\mu_1}\frac{\lambda_2}{\mu_2}\right)\right]^{-1} \tag{14}$$

The steady state availability of the system is derived as follows:

$$A_0 = P_0 + P_1 + P_2 + P_3\,. \tag{15}$$

The availability function in terms of decision parameters is described below:

$$A_0 = \left(1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_1}{\mu_1}\frac{\lambda_2}{\mu_2}\right)\left[1 + \left(1 + \frac{\lambda_1}{\mu_1}\right)\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_2}{\mu_2}\frac{\lambda_2}{\mu_2} + \frac{\lambda_2}{\mu_2}\frac{\lambda_3}{\mu_3}\right)\left(\frac{\lambda_1}{\mu_1}\frac{\lambda_1}{\mu_1}\frac{\lambda_2}{\mu_2}\right)\right]^{-1} \tag{16}$$

4 Numerical results and discussion

Nature-inspired algorithms (NIAs) are extensively utilized to find solutions and optimize performance of complex systems. NIAs are a group of efficient methodologies derived from natural activities, influenced, and inspired by biological phenomena. NIAs are also utilized to predict the optimum value of the operational availability of plants/ single unit systems in reliability engineering. These techniques have some flaws like slow convergence rate and being stuck in local optima. For this purpose, in present study, an efficient stochastic model is proposed for embedded life critical system is proposed. Reliability optimization of the embedded life critical system is also performed using NIA based algorithms namely Gray Wolf Optimization (GWO) and Dragon Fly (DA). These techniques are always recommended over traditional and well-known reliability evaluation techniques like regenerative point technique, cut set method, tie method and reliability block diagram approach to attain a global solution. The estimation of failure and repair rates is done. The simulation study is performed using R software on Windows 10 64-bit operating system having 8 GB of RAM and Intel Core i5 8th generation CPU. As the failure and repair rates are the decision variables which influences the availability of the embedded life critical system. The system has three failure rates $(\lambda_1, \lambda_2, \lambda_3)$ and three repair rates (μ_1, μ_2, μ_3). The range of these decision variables is provided in Table 1 as follows:

Table 1: Range of the decision variables

Sub-system	Range of failure-rate (η)	Range of repair-rate (δ)
Hardware	$\lambda_1 = [0.000003, 0.710]$	$\mu_1 = [0.000007, 2.20]$
Software	$\lambda_2 = [0.000005, 0.824]$	$\mu_2 = [0.000008, 2.35]$
Controller	$\lambda_3 = [0.000002, 0.920]$	$\mu_3 = [0.000009, 2.46]$

Table 2: Optimum availability of embedded life critical system with respect to iterations at various population size = 300 and 500 using GWO and DA

Iteration	Population size			
	300		500	
	GWO	DA	GWO	DA
40	0.9999992	0.9999992	0.9999992	0.9999992
60	0.9999992	0.9999992	0.9999992	0.9999991
80	0.9999992	0.9999992	0.9999992	0.9999992
100	0.9999992	0.9999992	0.9999992	0.9999981

Table 3: Optimum availability of embedded life critical system with respect to iterations at various population size = 700 and 900 using GWO and DA

Iteration	Population size			
	700		900	
	GWO	DA	GWO	DA
40	0.9999992	0.9999992	0.9999992	0.9999992
60	0.9999992	0.9999992	0.9999992	0.9999992
80	0.9999992	0.9999992	0.9999992	0.9999992
100	0.9999992	0.9999967	0.9999992	0.9999992

Table 4: Elapsed time of the GWO and DA algorithms used in attaining the optimum availability of embedded life critical system with respect to iterations at various population size = 300 and 500

Iteration	Population size			
	300		500	
	GWO	DA	GWO	DA
40	2.94	5.92	3	5.98
60	2.86	5.57	2.79	6.17
80	2.72	6.81	2.84	5.69
100	3.28	6.8	2.83	5.77

Table 5: Elapsed time of the GWO and DA algorithms used in attaining the optimum availability of embedded life critical system with respect to iterations at various population size = 700 and 900

Iteration	Population size			
	700		900	
	GWO	DA	GWO	DA
40	2.24	5.89	2.91	5.98
60	2.99	6.25	3.17	7.23
80	2.57	5.53	2.83	6.08
100	2.51	7.16	2.81	6.39

Table 6: Parameter estimation of various failure and repair rates after 40 iterations and different population sizes by using GWO and DA

Iter\NP		300	500	700	900
GWO	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	2.2
	μ_2	2.35	2.35	2.35	2.35
	μ_3	2.46	2.46	2.46	2.46
DA	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	1.978	2.2	2.2	2.2
	μ_2	0.889	2.35	2.35	2.35
	μ_3	2.46	2.46	2.46	2.46

Table 7: Parameter estimation of various failure and repair rates after 60 iterations and different population sizes by using GWO and DA

Iter\NP		300	500	700	900
GWO	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	2.2
	μ_2	2.35	2.35	2.35	2.35
	μ_3	2.46	2.46	2.46	2.47
DA	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	1.883803	2.2	2.2	2.2
	μ_2	2.35	0.06128191	2.35	2.35
	μ_3	2.46	2.168636	2.46	2.46

Table 8: Parameter estimation of various failure and repair rates after 80 iterations and different population sizes by using GWO and DA

Iter\NP		300	500	700	900
GWO	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	2.2
	μ_2	2.35	2.35	2.35	2.35
	μ_3	2.46	2.46	2.46	2.46
DA	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	2.2
	μ_2	2.35	2.35	2.35	2.35
	μ_3	2.46	2.46	2.46	2.46

Table 9: Parameter estimation of various failure and repair rates after 100 iterations and different population sizes by using GWO and DA

Iter\NP		300	500	700	900
GWO	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	2.2
	μ_2	2.35	2.349996	2.35	2.35
	μ_3	2.46	2.46	2.46	2.46
DA	λ_1	0.0000030	0.0000030	0.0000030	0.0000030
	λ_2	0.0000050	0.0000050	0.0000050	0.0000050
	λ_3	0.0000020	0.0000020	0.0000020	0.0000020
	μ_1	2.2	2.2	2.2	1.184561
	μ_2	2.35	2.35	0.1691405	2.35
	μ_3	2.46	1.040797	0.6154678	2.46

5 Conclusion

In present study a novel stochastic model is proposed for an embedded life critical system using the concept of cold standby redundancy for hardware and software components and controller/ operator failure. The global value of the system availability is derived using swarm-intelligence based algorithms GWO and DA. The availability is derived corresponding to various population sizes at different number of iterations. It is observed from Tables 2 and 3 that the optimal value of availability is 0.9999992 at 300, 500, 700, and 900 population sizes and 40 iterations. Though the elapsed time in execution of the program taken by GWO is very less in comparison to DA as shown in Tables 4-5. The estimated values of the parameters appended in tables 6-9. So, it is concluded that the global solution is derived only after 40 iterations at population size 300 and GWO performs better than the DA. In the literature, embedded critical systems reliability measures optimization is not such extensively explored. So, this work can be further extended to other optimization techniques for comparison purpose. Further, GWO and DA can be utilized to obtain the optimum availability of various process industries i.e., Paper and Pulp, Shoe Manufacturing, Sugar Industry, Sewage Treatment Plant, etc.

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